Q If α ~~and~~ represent three Dirac $\alpha_x, \alpha_y, \alpha_z$ and B and C are usual three dimensional vectors then show that

$$(\vec{\alpha}\cdot B)(\vec{\alpha}\cdot C) = B\cdot C + i\vec{\sigma}\cdot B\times C$$

Solution →

$$(\vec{\alpha}\cdot B)(\vec{\alpha}\cdot C) = (\alpha_x B_x + \alpha_y B_y + \alpha_z B_z)$$
$$\times (\alpha_x C_x + \alpha_y C_y + \alpha_z C_z)$$

$$= \alpha_x^2 B_x C_x + \alpha_y^2 B_y C_y + \alpha_z^2 B_z C_z$$
$$+ \alpha_x \alpha_y (B_x C_y - B_y C_x) + \alpha_y \alpha_z (B_y C_z - B_z C_y)$$
$$+ \alpha_z \alpha_x (B_z C_x - B_x C_z) \quad [\alpha_x \alpha_y = -\alpha_y \alpha_x]$$

$$= B_x C_x + B_y C_y + B_z C_z + \alpha_x \alpha_y (B_x C_y - B_y C_x)$$
$$+ \alpha_y \alpha_z (B_y C_z - B_z C_y) + \alpha_z \alpha_x (B_z C_x - B_x C_z)$$
$$(\text{Since } \alpha_x^2 = \alpha_y^2 = \alpha_z^2 = 1)$$

$$\alpha_x \alpha_y = \begin{bmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{bmatrix} \begin{bmatrix} 0 & \sigma_y \\ \sigma_y & 0 \end{bmatrix} = \begin{bmatrix} \sigma_x \sigma_y & 0 \\ 0 & \sigma_x \sigma_y \end{bmatrix}$$

$$= \begin{bmatrix} i\sigma_z & 0 \\ 0 & i\sigma_z \end{bmatrix} = i \begin{bmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{bmatrix} = i\sigma_z'$$

Similarly

$$\alpha_y \alpha_z = i\sigma_x' \text{ and } \alpha_z \alpha_x = i\sigma_z'$$

$$(\vec{\alpha}\cdot B)(\vec{\alpha}\cdot C) = (B\cdot C) + i\sigma_z'(B_x C_y - B_y C_x)$$
$$+ i\sigma_x'(B_y C_z - B_z C_y) + i\sigma_y'(B_z C_x - B_x C_z)$$

$$= B\cdot C + i\begin{bmatrix} \sigma_x' & \sigma_y' & \sigma_z' \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{bmatrix}$$

$$= B\cdot C + i\vec{\sigma}'\cdot B\times C$$

Q. If α and β are Dirac Matrices, Prove that

$$\alpha_x = \frac{1}{2}[\alpha_x \alpha_y, \alpha_y]$$

$$[\alpha_x \alpha_y, \alpha_y] = (\alpha_x \alpha_y \alpha_y - \alpha_y \alpha_x \alpha_y)$$

$$= \alpha_x \alpha_y^2 + \alpha_x \alpha_y \alpha_y$$

$$\boxed{\alpha_x \alpha_y = -\alpha_y \alpha_x}$$

$$[\alpha_x \alpha_y, \alpha_y] = \alpha_x \alpha_y^2 + \alpha_x \alpha_y^2$$

$$= \alpha_x + \alpha_x = 2\alpha_x$$

$$\boxed{\alpha_x = \frac{1}{2}[\alpha_x \alpha_y, \alpha_y]}$$